| Period | $T = \dfrac{1}{f}$ |
|--------|--------------------|
| Frequency | $f = \dfrac{1}{T}$ |

| Period | | Frequency | |
|--------|-----------|-----------|-----------|
| *Unit* | *Equivalent* | *Unit* | *Equivalent* |
| Seconds (s) | 1 s | Hertz (Hz) | 1 Hz |
| Milliseconds (ms) | $10^{-3}$ s | Kilohertz (kHz) | $10^{3}$ Hz |
| Microseconds (μs) | $10^{-6}$ s | Megahertz (MHz) | $10^{6}$ Hz |
| Nanoseconds (ns) | $10^{-9}$ s | Gigahertz (GHz) | $10^{9}$ Hz |
| Picoseconds (ps) | $10^{-12}$ s | Terahertz (THz) | $10^{12}$ Hz |

## Example 3.3

The power we use at home has a frequency of 60 Hz (50 Hz in Europe). The period of this sine wave can be determined as follows:

$$T = \frac{1}{f} = \frac{1}{60} = 0.0166 * 10^3 ms = 16.6 ms$$

This means that the period of the power for our lights at home is 0.0116 s, or 16.6 ms. Our eyes are not sensitive enough to distinguish these rapid changes in amplitude.

## Example 3.5

The period of a signal is 100 ms. What is its frequency in kilohertz?

**Solution**

First, we change 100 ms two seconds, and then we calculate the frequency from the period (1 Hz $=10^{-3}$ kHz).

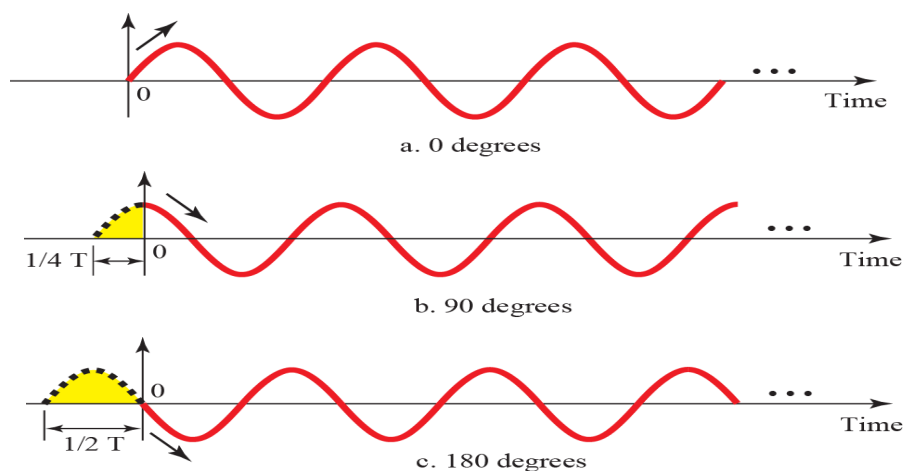ركزو بتحويل الوحدات

$$100ms = 100 * 10^{-3}s = 10^{-1}s$$

$$f = \frac{1}{T} = \frac{1}{10^{-1}} Hz = 10Hz = 10 * 10^{-3} kHz = 10^{-2} kHz$$

| **Phase :** is measured in degrees or radians | 360º is 2π rad<br>1º is 2π/360 rad<br>1 rad is 360/(2π) |
|---|---|



a. 0 degrees

b. 90 degrees

c. 180 degrees

Looking at Figure 3.6, we can say that
a. A sine wave with a phase of 0° starts at time 0 with a zero amplitude. The amplitude is increasing.
b. A sine wave with a phase of 90° starts at time 0 with a peak amplitude. The amplitude is decreasing.
c. A sine wave with a phase of 180° starts at time 0 with a zero amplitude. The amplitude is decreasing.
Another way to look at the phase is in terms of shift or offset. We can say that
a. A sine wave with a phase of 0° is not shifted.
b. A sine wave with a phase of 90° is shifted to the left by cycle. However, note that the signal does not really exist before time 0.
c. A sine wave with a phase of 180° is shifted to the left by cycle. However, note that the signal does not really exist before time 0.

## Example 3.6

A sine wave is offset $\frac{1}{6}$ cycle with respect to time 0. What is its phase in degrees and radians?

Solution

We know that 1 complete cycle is 360°. Therefore, $\frac{1}{6}$ cycle is

$$\frac{1}{6} * 360 = 60° * \frac{2\pi}{360} rad = \frac{\pi}{3} rad = 1.046 \, rad$$

| Wavelength (analog signal) | Wavelength = (propagation speed) * period $\frac{\text{propagation speed}}{\text{frequency}}$ $$\lambda = \frac{c}{f}$$ |
|---|---|

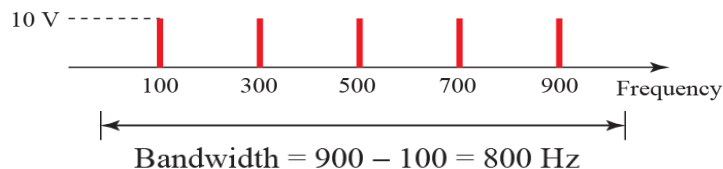| Bandwidth | $B = f_h - f_l$ | Hz |
|---|---|---|

## Example 3.10

If a periodic signal is decomposed into five sine waves with frequencies of 100, 300, 500, 700, and 900 Hz, what is its bandwidth?
Draw the spectrum, assuming all components have a maximum amplitude of 10 V.

Solution

Let $fh$ be the highest frequency, $fl$ the lowest frequency, and $B$ the bandwidth. Then

$$B= fh – fl = \ 900 - 100 = 800 \text{ Hz}$$



$$\text{Bandwidth} = 900 - 100 = 800 \text{ Hz}$$

Example 3.12

A nonperiodic composite signal has a bandwidth of 200 kHz, with a middle frequency of 140 kHz and peak amplitude of 20 V. The two extreme frequencies have an amplitude of 0. Draw the frequency domain of the signal.

Solution

The lowest frequency must be at 40 kHz and the highest at 240 kHz.
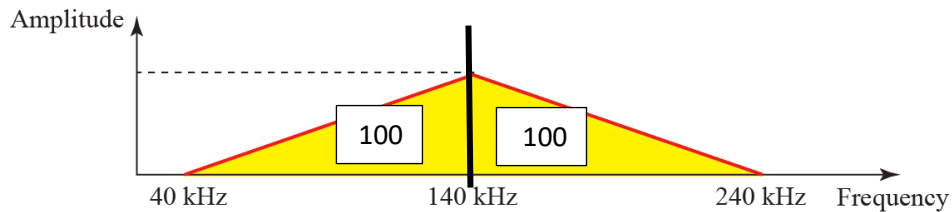
100+140=240(highest)
140-100=40 (lowest)



Figure 3.16 shows the frequency domain and the bandwidth.

| DIGITAL SIGNALS | Number of bits per level $=(log_2 level)$ bits |
|---|---|

**Example 3.16**

A digital signal has eight levels. How many bits are needed per level?

We calculate the number of bits from the following formula.
Each signal level is represented by 3 bits.
Number of bits per level $=(log_2 8)=$ 3 bits

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| Bit Rate | The bit rate is the number of bits sent in 1s (bps) |
|---|---|

Example 3.18

Assume we need to download text documents at the rate of 100 pages per second. What is the required bit rate of the channel?

Solution

A page is an average of 24 lines with 80 characters in each line. If we assume that one character requires 8 bits, the bit rate is

$$100 * 24 * 80 * 8 = 1,536,000 \text{ bps} = 1.536 \text{ Mbps}$$

| Bit Length (digital signal) | Bit length = propagation speed(سرعة) * bit duration (زمن) |
|---|---|

Decibel

| Attenuation | $\text{dB} = 10 log_{10} \dfrac{p_2}{p_1}$ |
|---|---|

Variables $P_1$ and $P_2$ are the powers of a signal at points

**Example 3.26**

Suppose a signal travels through a transmission medium and its power is reduced to one-half. This means that $P2 = P1$. In this case, the attenuation (loss of power) can be calculated as

$$10log_{10}\frac{p_2}{p_1}= 10log_{10}\frac{0.5\ p_2}{p_1}= 10log_{10}0.5 = 10(-0.3) = -3db$$

A loss of 3 dB ($-3$ dB) is equivalent to losing one-half the power.

---

<span style="color:red">Noise</span>            Signal-to-Noise Ratio (SNR)= $\dfrac{\textbf{average signal power}}{\textbf{average noise power}}$

$$SNR_{dB} = 10log_{10}SNR$$

---

**Example 3.31**

The power of a signal is 10 mW and the power of the noise is 1 µW; what are the values of SNR and SNRdB?

Solution

The values of SNR and SNRdB can be calculated as follows:

$$SNR = \frac{10.000\mu w}{1\mu w} = 10.000$$
$$SNR_{dB} = 10log_{10}10.000 = 10log_{10}10^4 = 40$$

---

Noiseless Channel: <span style="color:red">Nyquist Bit Rate</span>

$$Bitrate = 2*bandwidth*log_2L$$

طريقة حساب لوق باساس ٢ $(log_2x = \frac{log_{10}x}{log_{10}2})$

L is the number of signal levels used to represent data

---

**Example 3.34**

Consider a noiseless channel with a bandwidth of 3000 Hz transmitting a signal with two signal levels. The maximum bit rate can be calculated as

$$Bitrate = 2*bandwidth*log_2L$$
$$Bitrate = 2*300*log_2 2=6000bPs$$

**Example 3.36**

We need to send 265 kbps over a noiseless channel with a bandwidth of 20 kHz. <u>How many signal levels do we need?</u>

Solution

We can use the Nyquist formula as shown:

$$265.000 = 2*20.000*log_2L$$
$$log_2L=6.625$$
$$L=2^{6.625}=98.7\ levels$$

Since this result is not a power of 2, we need to either increase the number of levels or reduce the bit rate. If we have 128 levels, the bit rate is 280 kbps. If we have 64 levels, the bit rate is 240 kbps.

### Noisy Channel: Shannon Capacity

$$Capacity = bandwidth * log_2(1 + SNR)$$

### Example 3.37
Consider an extremely noisy channel in which the value of the signal-to-noise ratio is almost zero. In other words, the noise is so strong that the signal is faint. For this channel the capacity $C$ is calculated as
$$C = B * log_2(1 + SNR) = Blog_2(1 + 0) = Blog_2 1 = B * 0 = 0$$

This means that the capacity of this channel is zero regardless of the bandwidth. In other words, we cannot receive any data through this channel.

### Using Both Limits (Shannon & Nyquist)

### Example 3.41
We have a channel with a 1-MHz bandwidth. The SNR for this channel is 63. What are the appropriate bit rate and signal level?

Solution
First, we use the Shannon formula to find the upper limit.

$$C = B * log_2(1 + SNR) = 10^6 log_2(1 + 63) = 10^6 log_2 64 = 6Mbps$$

The Shannon formula gives us 6 Mbps, the upper limit. For better performance we choose something lower, 4 Mbps, for example. Then we use the Nyquist formula to find the number of signal levels.

$$4Mbps = 2*1MHz*log_2 L$$
$$L=4$$

### Throughput

### Example 3.44
A network with bandwidth of 10 Mbps can pass only an average of 12,000 frames per minute with each frame carrying an average of 10,000 bits. What is the throughput of this network?
Solution
We can calculate the throughput as

$$Throughput = \frac{12.000 * 10.000}{60} = 2Mbps$$

The throughput is almost one-fifth of the bandwidth in this case.

### Latency (Delay)

**Latency = propagation time + transmission time +queuing time + processing delay**

| | |
|---|---|
| *Propagation Time* | $$Propagation\ Time = \frac{distance}{Propagation\ speed}$$ |
| *Transmission Time* | $$Transmission\ Time = \frac{message\ size}{bandwidth}$$ |

## Example 3.45

What is the propagation time if the distance between the two points is 12,000 km? Assume the propagation speed to be $2.4 \times 108$ m/s in cable.

**Solution**

We can calculate the propagation time as

$$Propagation\ time = \frac{12.000 * 10.000}{(2.4 * 2^8)} = 50ms$$

The example shows that a bit can go over the Atlantic Ocean in only 50 ms if there is a direct cable between the source and the destination.

## Example 3.46

What are the propagation time and the transmission time for a 2.5-KB (kilobyte) message (an email) if the bandwidth of the network is 1 Gbps? Assume that the distance between the sender and the receiver is 12,000 km and that light travels at $2.4 \times 108$ m/s.

**Solution**

We can calculate the propagation and transmission time as

$$Propagation\ time = \frac{12.000 * 10.000}{(2.4 * 2^8)} = 50ms$$

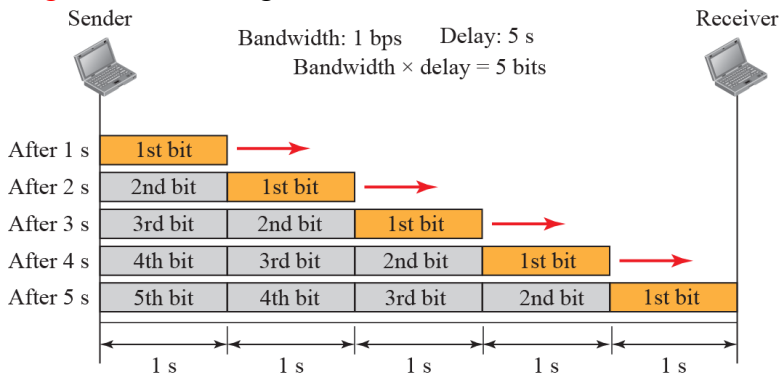$$Transmission\ time = \frac{2500 * 8}{10^9} = 0.020ms$$

Note that in this case, because the message is short, and the bandwidth is high, the dominant factor is the propagation time, not the transmission time. The transmission time can be ignored.

| | |
|---|---|
| Bandwidth-Delay Product | Bandwidth × delay |

**Case 1.** Figure 3.32 shows case 1.

Figure 3.32: Filling the links with bits for Case



**Case 2.** Now assume we have a bandwidth of 5 bps. Figure 3.33 shows that there can be maximum $5 \times 5 = 25$ bits on the line.

Figure 3.33: Filling the pipe with bits for Case 2.